New Approach for Hydraulic Design of Francis Runner Based on Empirical Correlations

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ABSTRACT

The design of Francis hydraulic turbine is a time consuming process even for experienced designers. This paper presents the process of design of the Francis type hydraulic turbines and also defines a function to approximate the perpendicular plan of the runner which provides a decrease in the design period. The results show a good agreement with the usual methods.

Keywords: Francis runner, hydroturbine, turbine design, meridional plan, perpendicular plan

1 INTRODUCTION

By ever-increasing use of hydroelectric energy and hydropower plants especially in developing countries, utilize of design methods which decrease the period of design process and also having the ability to design turbines as the base model for the optimization programs, is necessary. In this article, a new way for hydraulic design of Francis turbine based on empirical correlations is presented. This method not only is a part of runner design, but also it can be a good geometry for runner optimization process. The results show the advantages of this method for designing of the shape of runner, especially in respect of decreasing the design time.

The main procedure in design of a new runner includes: - classical theory for shaping the geometry – CFD analysis for the tuning of runner geometry

In the classical method, after the meridional plan of runner is designed based on available methods (here the BOVET method is used), by using of conformal mapping method, the perpendicular view of runner is obtained and then it will be modified.

2 MERIDIONAL PLAN

In the first step of classical design method of a Francis type hydraulic turbine, by the assumption of axial symmetry, the meridional view is obtained. The important dimensions of the runner are calculated by the BOVET. After the dimensions are determined, we can find the entrance and exit limit curves, that in our case the BOVET method has been used.

Fundamental parameters in turbine design are:

- H: Hydraulic head [m]
- Q: Turbine flow rate [m$^3$/sec]
- N: Turbine turns [rpm]

Regarding these parameters, the dimensionless specific speed $n_0$, can be defined as follows:

$$n_0 = \frac{2\pi N}{60(2gH)^{1/4}} \sqrt{\frac{Q}{\pi}}$$

which fixes the form of the runner channel.
The calculations in this article are based on BOVET method. The BOVET well known relations give the geometry of hub and shroud curves in the runner blade zone. The form of limit curves is defined by equation:

\[
\frac{y}{y_{ma}} = 3.08 \left(1 - \frac{r}{r_0}\right) \sqrt{\frac{r}{r_0}} \left(1 - \frac{r}{r_0}\right)
\]  \( (2) \)

To find the form and relative positions of two limit curves and also the value of 4 parameters in this equation, we should determine the presented values in the table 1. By using of BOVET relations, all these dimensions are relative respect to the runner hydraulic reference radius \( r_{2e} = 1 \).

<table>
<thead>
<tr>
<th>N(rpm)</th>
<th>Q(m3/sec)</th>
<th>H(m)</th>
<th>? (rad/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>750</td>
<td>1.8</td>
<td>95.7</td>
<td>78.5</td>
</tr>
<tr>
<td>( b_0 )</td>
<td>( b_0 )</td>
<td>( r_{0h} = y_{ma} )</td>
<td>( r_{0h} = r_{li} )</td>
</tr>
<tr>
<td>0.21</td>
<td>0.30</td>
<td>1.25</td>
<td>1.40</td>
</tr>
<tr>
<td>( l_1 )</td>
<td>( l_2 )</td>
<td>( x_{2e} )</td>
<td>( y_{2e} )</td>
</tr>
<tr>
<td>4.39</td>
<td>1.69</td>
<td>0.5</td>
<td>0.40</td>
</tr>
<tr>
<td>( y_{mi} )</td>
<td>( r_{mi} )</td>
<td>( v_{2e} )</td>
<td>( R_{2e} )</td>
</tr>
<tr>
<td>0.41</td>
<td>1.00</td>
<td>0.27</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Table.1

Comparing these results with real dimensions shows the truth of calculations. The real value of the outlet nominal radius \( R_{2e} \) is calculated with the formula:

\[
R_{2e} = \left( \frac{Q}{\pi \phi_{2e} \omega} \right)^{\frac{1}{2}}
\]  \( (3) \)

\[
R_{2t} = \left( \frac{2gh}{\omega h_{2t}} \right)^{\frac{1}{2}}
\]  \( (4) \)

Where \( \phi_{2e}, h_{2t} \) are numerical values which respectively, \( 0.24 \leq \phi_{2e} \leq 0.28 \) and \( 1.65 \leq h_{2t} \leq 1.8 \). (In this case \( \phi_{2e} = 0.27 \) and \( h_{2t} = 1.72 \).)
$R_{2e}$ and $R_{4e}$ only fix two points of the leading and trailing edges and the rest of these curves should be drawn to lead to better efficiency of runner. We use flow stream lines in the blade zone to locate the form and situation of these edges. To obtain the stream lines, the Poisson equation in cylindrical coordinate system $r, \theta, z$ will be considered:

$$\nabla \times \vec{v} = 0$$

$$\nabla \times \vec{v} = \left( \frac{1}{r} \frac{\partial v_r}{\partial \theta} - \frac{\partial v_\theta}{\partial z} \right) e_r + \left( \frac{\partial v_\theta}{\partial z} - \frac{\partial v_z}{\partial \theta} \right) e_\theta + \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r v_\theta \right) - \frac{1}{r} \frac{\partial v_r}{\partial \theta} \right] e_z$$

$$v_r = \frac{1}{r} \frac{\partial \psi}{\partial z}, \quad v_\theta = \frac{1}{r} \frac{\partial \psi}{\partial r}, \quad v_z = \text{const.}$$

$$\frac{\partial}{\partial z} \left( \frac{1}{r} \frac{\partial \psi}{\partial z} \right) - \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial \psi}{\partial r} \right) = 0$$

$$\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r^2} \frac{\partial \psi}{\partial r} + \frac{1}{z^2} \frac{\partial^2 \psi}{\partial z^2} = 0 \quad (5)$$

The above equation is solved by Finite Difference Method, considering the following boundary conditions:

In entrance: $v_z = \frac{1}{r} \frac{\partial \psi}{\partial z} = 0$

In exit: $v_r = -\frac{1}{r} \frac{\partial \psi}{\partial r} = 0$

![Fig. 2. Flow stream lines pattern](image)

With assumption $\psi = \text{const.}$ through a stream line and $\psi = 0, \psi = 1$ for channel walls and solving the Poisson equation, we try to find the stream lines for $\psi = 0.1, 0.2, \ldots, 0.9$ because $\psi = 0, \psi = 1$ are specified for solid wall curves.

After these stream lines are determined and knowing the location of fixed points $2e, 1e$, we can estimate the form of leading and trailing edges by assuming equal flow rate for each of flow channels:

$$\Delta Q = 2\pi R \cdot \Delta \theta \cdot v = \text{const.}$$

$$\Delta \theta \cdot R = \text{const.}$$
In another way, the field of velocity and pressure along stream lines can be determined and with the values of the velocities and pressure in the direction of the fluid, it would be possible to find useful results for design of the entrance and exit of turbine runner.

Fig.4. Velocities variation along stream line \((n_s=97)\)

In this case, the form of these edges is two parabolic curves.
In the entrance for specific speeds between \(0.089 \leq n_{0} \leq 0.275\), the leading edge form is a parabolic arc with the peak in the point by radius of \(2r_1 - r_3\) which passes through the points J and B, and for specific speeds between \(0.275 \leq n_{0} \leq 0.89\) its form is also a parabolic arc but with the minimum point in the J and the axis is parallel to runner axis.
In the exit area, trailing edge is a parabolic curve which has a minimum point in I and also passes through a point such as X with a radius of \(r_b/3\).

Fig.5. Parabolic leading and trailing edges
3 PERPENDICULAR PLAN

When the flow stream lines between the entrance and exit edges are determined, other turbine specifications such as entrance and exit flow angles can be calculated. Then we can use the conformal mapping method to draw the form of blade in the perpendicular plan.

This method is based on trial and error, and gives us the ability to map each of the stream lines from meridional view to the perpendicular plan.

A curved triangle can be drawn by known stream lines and its flow angles. This triangle is a transformation link between two plants.

By assuming a point on the perpendicular view with radius $R$, the position of this point on this view is determined

$$\Delta A_t = R \cdot \Delta \varphi$$  \hspace{1cm} (7)

So, from the length $\Delta A_t$ on the correspondent triangle, we can estimate its projection $\Delta x_t$ on the meridional plan.

Then on the meridional plan, by moving along the stream line equal to $\Delta x_t$, the radius of new point in the meridional view can be estimated and then the new radius is compared with assumed one. This trial and error loop will continue until obtaining the best result.

Regarding decreasing the design period in this part, we can estimate the curved edge of triangle as a second order polynom and each of the three unknown of this function can be calculated:

$$f(x) = ax^2 + bx + c$$

$$f(0) = 0 \Rightarrow c = 0$$

$$(@ x = 0) \Rightarrow \frac{df}{dx} = 2ax + b = \tan \beta_2 \Rightarrow b = \tan \beta_2$$

$$(@ x = A_{max}) \Rightarrow \frac{df}{dx} = 2ax + b = \tan \beta_2 \Rightarrow a = \frac{\tan \beta_1 - \tan \beta_2}{2A}$$

$$f(x) = \left[\frac{\tan \beta_1 - \tan \beta_2}{2A}\right]x^2 + x\tan \beta_2$$  \hspace{1cm} (8)

$$L_m = \left[\frac{\tan \beta_1 - \tan \beta_2}{2} + \tan \beta_2\right] A \Rightarrow A = \frac{2L_m}{\tan \beta_1 + \tan \beta_2}$$  \hspace{1cm} (9)

So, the value of $A$ is determined, and without using of trial and error loop and in a shorter period of transformation, the perpendicular plan of blade will be obtained. The radius of each point on the meridional plan is known and directly by using this value on the triangle, the distance of this point on the perpendicular plan will be resulted.

In the last step of design process, the 3D image of the whole runner can be drawn using the two plans.
REFERENCES

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